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Turbulent onset in moderately large convecting layers

R. P. Behringer
Department of Physics, Duke University, Durham, North Carolina 27706

J. N. Shaumeyer,* C. A. Clark, and C. C. Agosta*
Department of Physics, Wesleyan University, Middletown, Connecticut 06457
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We present long-time thermal data on turbulent evolution in Rayleigh-Bénard convection for cylindrical containers of aspect ratios $\Gamma = 6.22$ and 7.87 and Prandtl numbers near 0.6. The first time dependence observed was more complex than the intermittent flows reported by Ahlers and Walden for $\Gamma = 4.72$, although the periodicity they observed was reproduced for $\Gamma = 7.87$. The turbulent onset for $\Gamma = 6.22$, showing substantial regimes of periodicity, was quite different from that for $\Gamma = 7.87$ or 4.72. We conclude that changes of order unity in Γ strongly affect turbulent onset, even for moderately large aspect ratios.

Rayleigh-Bénard convection occurs when the temperature difference ΔT across a fluid layer exceeds a critical value ΔT_c. Recent studies[1–5] of convection have probed the onset of turbulence, the subject we address in this work. It is known[2,4] that the transition to turbulence is strongly influenced by the aspect ratio Γ, where Γ is a measure of the horizontal dimension L of the fluid in units of its height d. For our measurements, made in cylindrical containers, the appropriate choice for L is the radius, and we report measurements for $\Gamma = 7.87$ and 6.22. These aspect ratios exceed the small-aspect-ratio limit, $\Gamma \lesssim 3$, for which the onset of turbulence is usually characterized by deterministic flows related to models[6] with only a few degrees of freedom. However, studies of the onset of chaos for larger Γ are of both theoretical and practical interest. Our experiments and those by Ahlers and Walden[3] for $\Gamma = 4.72$, which all use normal liquid ^{4}He as the fluid, show a complex onset of turbulence, but certain features may indicate systematic behavior.

In addition to Γ, the remaining parameters for a Boussinesq system are the Prandtl number $P = \nu/\kappa$, the Rayleigh number $R = g\alpha_p d^3 \Delta T/\kappa\nu$, and the thermal diffusion time $t_v = d^2/\kappa$. The parameters ν, κ, α_p, and g are defined conventionally.[4] Convection begins at R_c, and $r \equiv R/R_c$ is a convenient parameter.

Our data consist of long-time measurements of ΔT versus time t with the heat current q and the temperature at the top of the layer held fixed. When time dependence sets in, fluctuations δT_b in ΔT are observed at the bottom of the layer. We find that the set of events leading to turbulence is qualitatively different for $\Gamma = 7.87$ vs $\Gamma = 6.22$. Our data for $\Gamma = 7.87$ do have some similarities to the results of Ahlers and Walden[3] for $\Gamma = 4.72$.

We show in Figs. 1 and 2 data for $\Gamma = 7.87$ ($P = 0.61$), where the number to the right of each run is the time average of r. Most runs were obtained by increasing q by small steps, roughly one percent of the critical value, until the desired value was obtained. Runs a through i were obtained sequentially in this fashion. However, runs j, l, and

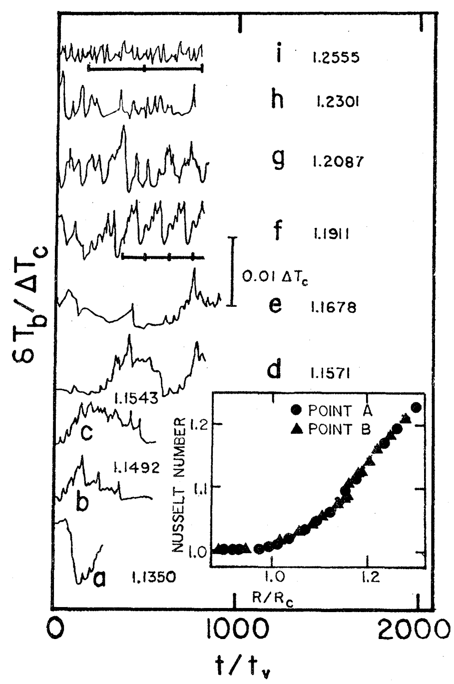

FIG. 1. Fluctuations δT_b in the temperature at the bottom of the fluid layer for $\Gamma = 7.87$ vs time in units of t_v. The number to the right of each run is the time-averaged value of the reduced Rayleigh number $r = R/R_c$. Regions showing some periodicity are underlined and the periods are marked by vetical bars. Inset: The Nusselt number N vs r for $\Gamma = 7.87$ at two different points in the ^{4}He phase diagram.

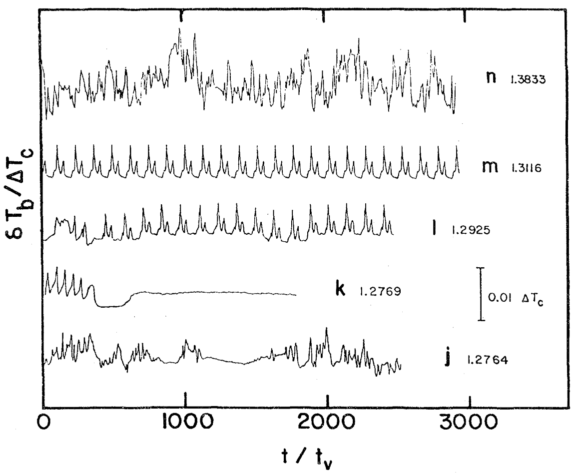

FIG. 2. A continuation of the results shown in Fig. 1 for $\Gamma = 7.87$.

n were obtained by increasing q in larger steps. Run k was obtained from run j and run m was obtained from run l.

For R just above R_c, our flows have immeasurably small time dependence, after initial transients have decayed. Specifically, for $1.0 < R/R_c < 1.135$ and times as long as $100 t_v$, any fluctuations in ΔT were smaller than $3\times 10^{-5}\Delta T_c$. Before the time dependence becomes recognizable as the steady turbulence reported previously[4,5] there is a region with $r \simeq 1.30$ of periodic oscillations (l and m). Immediately preceding the oscillations the flow is quiescent (k). The fundamental frequency of the periodic regime differs little from the frequencies $\int |\omega| P(\omega)\, d\omega / \int P(\omega)\, d\omega$ derived[5] from the power spectra $P(\omega)$ for broadband turbulence at slightly higher Rayleigh numbers. Runs k, l, and m are strikingly similar to states observed by Ahlers and Walden[3] near $1.9R_c$. The fundamental frequency of our run m was $t_v\omega = 0.049$; the frequency of run j reported by Ahlers and Walden was 0.24. To our knowledge, no steady periodic motions of this frequency have been predicted. For instance, the oscillatory instability predicted by Clever and Busse[7] has $t_v\omega \gtrsim 20$ for $P = 0.6$.

We do not, however, observe the intermittency reported by Ahlers and Walden. Instead, our data show considerably more structure close to R_c, with occasional hints of periodicity. In runs f and i there are, respectively, three and two cycles of complex periodic structures, as indicated by the underlines and vertical bars.

Our data also differ from that of Ahlers and Walden in the heat transport. A dimensionless measure of this quantity is the Nusselt number,[8] N, and our results for $N(r)$ with $\Gamma = 7.87$ are given in the inset of Fig. 1. In addition to the Nusselt values corresponding to Figs. 1 and 2 (operating point A) we show data (operating point B) obtained with the same Γ and P but at a different point in the ^{4}He phase diagram. Both operating points show a change in slope from $S = 0.54$ to 0.93 at $r \simeq 1.14$, indicating a transition in the planform of the flow. Run a of Fig. 1 was just at this transition and is evidently time dependent. Unfortunately, it is not possible to observe the flow patterns in our experiments. But we can rule out the inverted bifurcation from hexagons to rolls which has been studied theoretically by Busse[9] and experimentally by Walden and Ahlers.[10] This transition is associated with departures from the Oberbeck-Boussingesq approximation as given by a parameter $Q(R)$.[10] For $\Gamma = 7.87$, $Q(R_c)$ takes on the values -0.439 and -0.082, respectively, for points A and B. Thoery and experiment agree that even for point A all hexagon-roll transitions would occur below $r - 1 \simeq 3\times 10^{-3}$. Ahlers *et al.*[11] have calculated slopes for various flow patterns finding $S = 0.60$, 0.72, 0.45, and 0.91 for patterns of hexagons, straight rolls, concentric nodeless roles, and concentric rolls with a central node. Our slope of 0.54 may indicate hexagons or nodeless rolls and our slope of 0.93 may indicate straight rolls or concentric rolls with a node. We note that Behringer and Ahlers[8] reported $S = 0.83$ for the stable convective state for $\Gamma = 4.72$, and a metastable state of $S = 0.56$.

We now turn to the evolution of time dependence for $\Gamma = 6.22$ ($P = 0.55$) as shown in Figs. 3 and 4. For these data $Q(R_c) = -1.3\times 10^{-3}$ and no slope changes were observed in $N(r)$.[12] Runs a, b, d, f, g, and h were initiated by taking small steps from a subcritical state, whereas g–r (excluding h) were obtained sequentially by small steps. Run c was obtained from b, and run e from d. The onset of turbulence in this case differs significantly from the previous situation. The first time dependence is a periodic flow superposed over very slow background motion. The fundamental frequency is $t_v\omega_1 = 0.037$ at $1.155R_c$, and the frequency approximately triples by $1.189R_c$. The periodicity observed in runs a–c was found to be reproducible, and ends in run d by a gradual slowing down of the oscillations. Another periodic state occurs at $1.42R_c$ with frequency $t_v\omega_2 = 0.23$, but this state has lost coherence at $1.454R_c$. An $800t_v$ oscillatory region of frequency ω_3 occurs at $1.514R_c$, showing a characteristic double-peak–double-minimum form. This structure is easily seen in a power spectrum because the fundamental frequency has about ten times less power than the second harmonic. The double-peaked structure is also identifiable in a number of the remaining runs, as indicated by the underlined sections, and exists without interlude in runs p and q. We find $t_v\omega_3 = 0.199(r - 1.074)$. Run r shows stretches of periodicity buried in a chaotic background with $t_v\omega_3 = 0.136$, where 0.136 corresponds to the double-peak structure. We also find segments at $\frac{1}{2}$ and $\frac{1}{4}$ this frequency as shown along

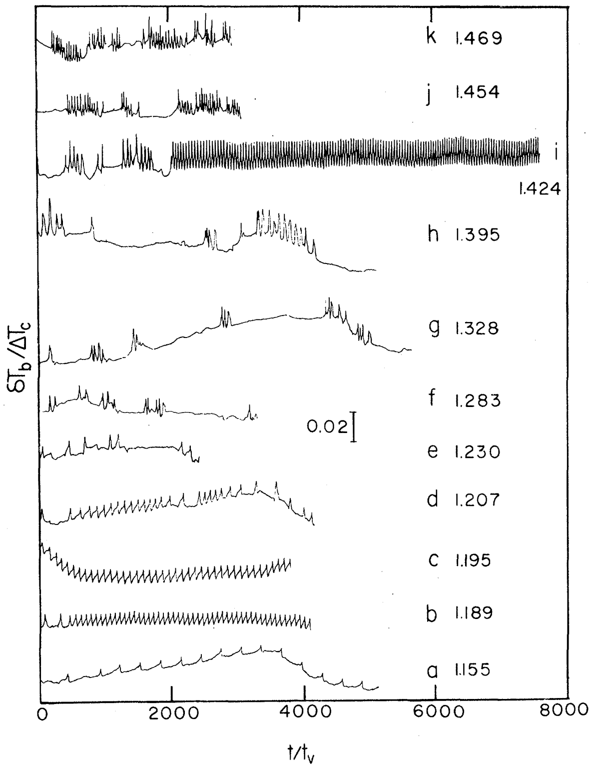

FIG. 3. Data similar to those in Figs. 1 and 2, but with $\Gamma = 6.22$.

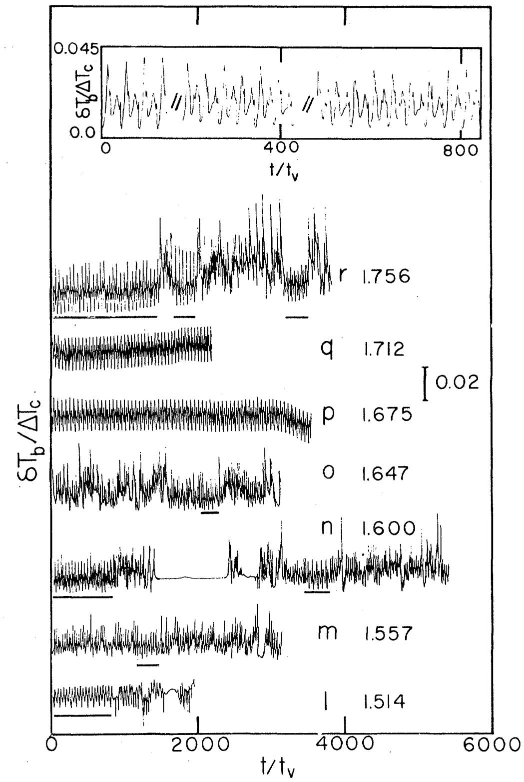

FIG. 4. A continuation of Fig. 3 showing data similar to Figs. 1 and 2, but with $\Gamma = 6.22$. The inset shows sections of run r with, from left to right, two-, four-, and eight-peak structures.

with the double-peaked oscillations in the insert of Fig. 4. The following run at 1.793 (not shown) does not have any periodic segments; its spectrum is broadband and falls off as ω^{-a} where $a = 3.5 \pm 0.5$.

To conclude, we have presented data showing the evolution of turbulence for $\Gamma = 6.22$ and 7.87. These data do not show the kind of intermittency reported by Ahlers and Walden for $\Gamma = 4.72$. Instead, more complex structures appear in the time dependence closest to R_c. However, the robustly periodic state, run m, for $\Gamma = 7.87$ strongly resembles run j found for $\Gamma = 4.72$. The data for $\Gamma = 6.22$ show a number of periodic bands in R/R_c, but there is little resemblance to the results for $\Gamma = 7.87$ and 4.72. On balance, changes of order unity in Γ have a strong effect on the nature of turbulent onset, even when Γ is moderately large as in our experiments. Our data are neither consistent with a gradual variation in the route to chaos as Γ increases, nor are they describable by simple models. Nevertheless, there are a number of intriguing periodic regimes suggesting the possibility of systematic behavior. An important question which remains to be answered is the effect on turbulent onset by aspect-ratio changes much smaller than unity. Such an experiment might help reveal changes in the model structure which may be responsible for the differing approaches to turbulence observed in the present and previous work.

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*Current address: Physics Department, Duke University, Durham, N.C. 27706.

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[12]The slope of $N(r)$ obtained for $\Gamma = 6.22$ was 0.62. The value is lower than the result $S = 0.83$ obtained for $\Gamma = 4.72$, and the large r slope of 0.93 obtained for $\Gamma = 7.87$. It seems likely that the results for turbulent onset for $\Gamma = 6.22$ were obtained in a different flow state than those for $\Gamma = 4.72$ and 7.87.